



BOGIES WITH RADIAL STEERING AXLES RUNNING IN CURVES

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Summary: *Knowing that when running in curves the position of the axles of the bogies with radial steering axles are very close to the radial position under the action of the tangential contact forces, the present study analyses the technical possibilities to ensure the pure rolling of a vehicle running in a curve. Based upon a kinematics study of the axle in order to determinate the creep speed between the wheels and the rails, by using Kalker's theory, the friction coefficients and the tangential contact forces are determined. Moreover, it was analyzed the possibility of pure for a vehicle running in curves with radiuses as small as possible.*

1. Introduction

When running in curves the position of the axles of the bogies with radial steering axles are very close to the radial position under the action of the tangential contact forces.

If the bearings of the axles of a bogie are fastened with elastic elements, the very close to radial position of the leading axle involves the decrease of the transversal creep while the conical rolling determines the vanishing of the longitudinal creep. The result in named pure rolling.

The technical possibilities to ensure the pure rolling of a vehicle running in a curve are the object of the present study.

The paper presents facts regarding the kinematics of the axle in order to determinate the creep speed between the wheels and the rails.

Moreover, using Kalker's theory, the friction coefficients and the tangential contact forces are determined.

The influences of the axles guiding system and that paper is focused on the possibility of pure rolling for a vehicle running in curves with radiuses as small as possible.

2. Kinematics of a Bogie Running in a Curve

It is considered a bogie with two guided axles and wheels with wear profile running with a speed v in a curve of radius R .

In the general case, when the vehicle forward speed differs from the normal velocity of the curve, which implies different loads on the wheels, the creep velocities of the external wheels w_e are different from the creep velocities w_i of the interior of the curve.

In the given by thesis, the creep speeds at the external wheel w_e and at the internal wheel w_i are:

$$(1) \quad w_{ex} = -w_{ix} = w_x = (e/R - \gamma \cdot y_c / r) \cdot v;$$

$$(2) \quad w_{ey} = w_{iy} = w_y = -v \cdot \alpha,$$

where:

- y_c is the displacement of the axle from the median position in the track;
- e is the half-distances between the rolling circles;
- r is the rolling radius for an axle in median position;
- γ in the angle of the cone wheel of the axle;
- α is the angle between meridian plane of the axis of the track.

The expression of the coefficient K is:

$$(3) \quad K = (s/R) \cdot (R + \eta) / (s + \eta),$$

where $s = e \cdot r / (\gamma \cdot y_c)$ and $\eta = (\Delta Q / Q_0) \cdot e$ is a term which depends on the axle load transfer ΔQ , Q_0 being the static wheel load.

As it results from (1) and (2), for $K = 1$, the conical rolling is realized when the longitudinal creep speed $w_x = w_y$ for a displacement $y_c = 0$, is obtained for an angle $\alpha = 0$, when the axle has a radial position.

3. Position of the Bogie's Axles in Curves

The position of each axle in curve is determined by the forces and torques, which act over them. Thus, in the wheel-rail contact, in the case of insignificant y_c discrepancies, according to Kalker's theory [2] the friction coefficients are:

$$(4) \quad \tau_x = \chi_x \cdot v_x \quad \text{and} \quad \tau_y = \chi_y \cdot v_y + \chi_s \cdot v_s$$

while the corresponding tangential forces are:

$$T_x = v_x \cdot Q; \quad T_y = v_y \cdot Q.$$

The two relation debated in point (4) are using the following coefficients:

$$v_x = w_x / v, \quad v_y = w_{yx} / v, \quad v_s = r \cdot \omega_s / v$$

represent the values of the longitudinal, transversal and spin creep, while ω_s is the angular spin velocity:

$$\omega_s = K \cdot (v / r) \cdot \gamma.$$

Q represents the load distributed on each wheel, as it follows:

$$Q = Q_0 + \Delta Q_0, \text{ for the external wheel;}$$

$$Q = Q_0 - \Delta Q_0, \text{ for the internal wheel.}$$

The axle load transfer ΔQ_0 depends upon the transversal acceleration value, which is determined by the vehicle's suspension and the running speed in the curve. Thus, if the vehicle is running with the normal velocity of the curve, each wheel is loaded with the same force Q . The creep coefficients are also defined as it follows:

$$\chi_x = (G \cdot a \cdot b / Q) \cdot C_{11};$$

$$\chi_y = (G \cdot a \cdot b / Q) \cdot C_{22};$$

$$\chi_s = [G \cdot (a \cdot b)^2 / Q] \cdot C_{23}.$$

where:

- a, b stand for the half axes of the contact ellipsis determined by the use of Hertz's theory;
- G is the elastic transversal module of steel;
- C_{11}, C_{22}, C_{23} are the coefficients calculated by Kalker depending on the rate a/b .

Chartet's approximation [1] were also used in order to define the friction coefficients as bellow:

$$(5) \quad \tau_x = \mu \cdot v_x / \sqrt{(\mu / \chi)^2 + \vartheta^2};$$

$$(6) \quad \tau_y = \mu \cdot v_y / \sqrt{(\mu / \chi)^2 + \vartheta^2},$$

in which μ represents the coefficient of adherence, and

$$(7) \quad \chi = (G \cdot a \cdot b / Q) \cdot (C_{11} + C_{22}) / 2.$$

The horizontal components of the forces between the wheels and the rails of the two axles are also taken into account:

$$(8) \quad C = c_g \cdot y_c = [2 \cdot Q \cdot \gamma / (\rho_r \cdot \gamma_0)] \cdot y_c.$$

In the previously discussed equation, c_g is the elasticity constant, ρ_r is the curvature radius of the wheel profile in the contact point while γ_0 stands for the angle

between the rolling surface and the horizontal plane in the contact point where the axle center is situated in the middle of the track.

The elastic elements of the guiding system of the axle are considered to have linear characteristics. The external forces, which acts over bogie, is considered to have a constant value being transversally applied in its mass center.

The determined expressions of the forces are used in the quasistatic equations of the bogie running in a curve determining the angle α of the two axles and their displacements y_c for different values of the curve, running velocity and elastic constants in the guiding system of the axles.

4. Movement Equations of the Bogie

Considering $f_x = \chi_x \cdot Q$ and $f_y = \chi_y \cdot Q$, the pseudocreep forces of the first and the second bogie's axles are:

$$(9) \quad T_{1x} = f_x \cdot v_{1x} = f_x \cdot (\gamma / r \cdot y_{c1} - e / R)$$

$$(10) \quad T_{1y} = f_y \cdot v_{1y} = f_y \cdot \alpha_1$$

$$(11) \quad T_{2x} = f_x \cdot v_{2x} = f_x \cdot (\gamma / r \cdot y_{c2} - e / R)$$

$$(12) \quad T_{2y} = f_y \cdot v_{2y} = f_y \cdot \alpha_2$$

In addition, the centering forces for two axles would be:

$$C_1 = c_g \cdot y_{c1} \quad \text{and} \quad C_2 = c_g \cdot y_{c2}$$

From the equilibrium conditions for forces and torques, acting on the bogie it was obtained the mathematical representation of the inscribing in curves phenomenon:

$$(13) \quad \begin{aligned} & c_g \cdot y_{c1} - 2 \cdot c_y \cdot (y_1 - y_{c1}) + 2 \cdot f_y \cdot \alpha_1 = 0 \\ & 2 \cdot e \cdot f_x \cdot (\gamma / r \cdot y_{c1} - e / R) - 2 \cdot c_x \cdot b^2 \cdot [\alpha_1 + a / (2 \cdot R) + (y_1 - y_2) / a] = 0 \\ & c_g \cdot y_{c2} - 2 \cdot c_y \cdot (y_1 - y_{c2}) + 2 \cdot f_y \cdot \alpha_2 = 0 \\ & 2 \cdot e \cdot f_x \cdot (\gamma / r \cdot y_{c2} - e / R) - 2 \cdot c_x \cdot b^2 \cdot [\alpha_2 - a / (2 \cdot R) + (y_1 - y_2) / a] = 0 \\ & 2 \cdot c_y \cdot (y_1 - y_{c1}) + 2 \cdot c_y \cdot (y_2 - y_{c2}) - F_n = 0 \\ & 2 \cdot c_x \cdot b^2 \cdot [\alpha_1 + a / (2 \cdot R) + (y_1 - y_2) / a] + \\ & + 2 \cdot c_x \cdot b^2 \cdot [\alpha_2 - a / (2 \cdot R) + (y_1 - y_2) / a] + \\ & + 2 \cdot c_y \cdot (y_1 - y_{c1}) - 2 \cdot c_y \cdot (y_2 - y_{c2}) = 0 \end{aligned}$$

where:

- c_x and c_y are the axles conducting system's longitudinal and transversal rigidities;
- y_1 and y_2 the bogie's longitudinal axle displacement in comparison with the railway's axle, corresponding to the two axles;
- α_1 and α_2 the axles attack angle;
- a the bogie's wheelbase;

- $2b$ the transversal distance between the axle's suspension springs;
- F_n the lateral force acting in the center of bogie.

Considering $c_g = 0$, from the equation system (13) results:

$$(14) \quad y_{c1} = y_{c0} \cdot \left(1 + \frac{f_y}{f_x} \cdot \frac{a^2}{4 \cdot e^2} \cdot \frac{A}{C} \right) + \frac{a}{2} \cdot \frac{F_n}{4 \cdot f_y} \cdot \frac{B}{C}$$

where:

$$A = 1 + \frac{f_x \cdot a}{2 \cdot c_x \cdot b^2} \cdot \frac{e \cdot \gamma}{r} \cdot \left(1 + \frac{4 \cdot c_x \cdot b^2}{c_y \cdot a^2} \right);$$

$$B = f_y \cdot a / (2 \cdot c_x \cdot b^2) - 1;$$

$$C = 1 + \frac{f_x \cdot f_y \cdot a^2}{(2 \cdot c_x \cdot b^2)^2} \cdot \frac{e \cdot \gamma}{r} \cdot \left(1 + \frac{4 \cdot c_x \cdot b^2}{c_y \cdot a^2} \right)$$

and

$$y_{c0} = e \cdot r / (\gamma \cdot R)$$

The first term of the relation (13) shows the deviation from the railway axis and the second one represents the radial axle's displacement due to the lateral forces F_n .

For small elasticity of the suspension, the deviation from the railway axis increases in comparison with the fix axles and if $2 \cdot c_x \cdot b^2 > f_x \cdot a$, then the suspension's elasticity does not improves the vehicle's inscribing into the curve.

If $2 \cdot c_x \cdot b^2 < f_x \cdot a$, than the deviation from the railway axis decreases and for a well enough elastic suspension would become close enough to the minimum possible $y_{c1} = y_{c0}$.

When $2 \cdot c_x \cdot b^2 > f_x \cdot a$, the deviation may also decrease by increasing the value of the term $e \cdot \gamma / r \cdot [1 + 4 \cdot c_x \cdot b^2 / (c_y \cdot a^2)]$, which means either by a big effective conicity γ or by decreasing the transversal elasticity c_y . the displacement from the railway axis of the second axle is:

$$(15) \quad y_{c2} = y_{c0} \cdot \left(1 + \frac{f_y}{f_x} \cdot \frac{a^2}{4 \cdot e^2} \cdot \frac{2 - A}{C} \right) + \frac{a}{2} \cdot \frac{F_n}{4 \cdot f_y} \cdot \frac{B + 2}{C}$$

For $F_n = 0$, the angles between the axles and the normal to railway are:

$$(16) \quad \alpha_1 = -\alpha_2 = -(1/C) \cdot a / (2 \cdot R)$$

The maximum pseudocreep force would appear to the two front wheels. In their case, from $F_n = 0$, the pseudocreep are:

$$(17) \quad v_{1x} = \frac{\gamma}{r} \cdot \left(1 + \frac{f_y}{f_x} \cdot \frac{a^2}{4 \cdot e^2} \cdot \frac{A}{C} \right) \cdot y_{c0} - \frac{e}{R} = \frac{f_y}{f_x} \cdot \frac{a^2}{4 \cdot e \cdot R} \cdot \frac{A}{C}$$

$$(18) \quad v_{1y} = \frac{1}{C} \cdot \frac{a}{2 \cdot R}$$

and the corresponding force, considering $f_x = f_y = f$, is:

$$(19) \quad T = \sqrt{(f_x \cdot v_x)^2 + (f_y \cdot v_y)^2} = \frac{f \cdot a}{2 \cdot R \cdot C} \cdot \sqrt{1 + \frac{a^2}{4 \cdot e^2} \cdot A^2} \leq \mu \cdot Q$$

This means that a bogie having elastic conducted axles would slide in any curve having a radius

$$R \leq \frac{f \cdot a}{2 \cdot C \cdot \mu \cdot Q} \cdot \sqrt{1 - \frac{a^2}{4 \cdot e^2} \cdot A^2}.$$

The more elastic the suspension is, the smaller the curve's radius R would be, the axles tending to a radial position.

Comparing the displacement due to the lateral force F_n one may see that the second axle is more displaced than the front axle, the displacement being independent both of the curve's radius and of the deviation from the railroad axes. The deviation exists even in the absence of any lateral forces, aspect indicating the inner capacity of the bogie for self-guiding the pseudocreep forces between the wheels and the rails.

5. Calculus. Example

Data for the calculus: we shall consider a vehicle AVA 200 with Y32 bogies.

- the transversal stiffness $c_y = 3,8 \cdot 10^6 \text{ N/m}$
- the longitudinal stiffness $c_x = 6 \cdot 10^5 \text{ N/m}$
- the distance between the rolling circles $2e = 1,5 \text{ m}$
- the transversal distance between the axles suspension springs $2b = 2 \text{ m}$
- the effective conicity $\gamma = 0,15$
- the wheel radius for the rolling circle $r = 0,460 \text{ m}$
- the curvature radius of the wheel $\rho_r = 0,5 \text{ m}$
- the curvature radius of the bogie $\rho_s = 0,3 \text{ m}$
- the wheelbase of the bogie $a = 2,56 \text{ m}$
- the vehicle mass $m = 27500 \text{ kg}$
- the mass of the bogie $m_{sb} = 2500 \text{ kg}$

The wheel load is given by the relation:

$$Q = \frac{(m + 2m_{sb}) \cdot g}{8} \quad Q = 3.985 \cdot 10^4 \text{ N}$$

The pseudocreep coefficient is given by the relation

$$\chi = \frac{400}{\sqrt[3]{Q}}, \text{ where } Q \text{ should be in kN.}$$

It results: $\chi = 117,104$, $\chi_x = \chi_y = \chi$, $f_x = f_y = f$, $f = \chi \cdot Q$, $f = 4,868 \cdot 10^6$.

For this study we shall consider a curve radius $R = 350 \text{ m}$.

In order to simplify the torque equations we shall consider $c_g = 0$ and the lateral forces acting over the bogie $F_n = m \cdot \gamma_{T0}$, $F_n = 0$.

The axle's displacement from the median position:

$$y_{c0} = \frac{e \cdot r}{\gamma \cdot R} \quad y_{c0} = 0,006 \text{ m}$$

$$A = 1 + \frac{f_x \cdot a}{2 \cdot c_x \cdot b^2} \cdot \frac{e \cdot \gamma}{r} \left(1 + \frac{4 \cdot c_x \cdot b^2}{c_y \cdot a^2} \right) \quad A = 3,67$$

$$B = \frac{f_y \cdot a}{2 \cdot c_x \cdot b^2} - 1 \quad B = 8,956$$

$$C = 1 + \frac{f_x \cdot f_y \cdot a^2}{(2 \cdot c_x \cdot b^2)^2} \cdot \frac{e \cdot \gamma}{r} \cdot \left(1 + \frac{4 \cdot c_x \cdot b^2}{c_y \cdot a^2} \right) \quad C = 27,579$$

Considering the prevision relations for the coefficients, we may establish the deviations of the axles:

$$y_{c1} = y_{c0} \cdot \left(1 + \frac{f_y}{f_x} \cdot \frac{a^2}{4 \cdot e^2} \cdot \frac{A}{C} \right) + \frac{a}{2} \cdot \frac{F_n}{4 \cdot f_y} \cdot \frac{B}{C} \quad y_{c1} = 0,009 \text{ m}$$

$$y_{c2} = y_{c0} \cdot \left(1 + \frac{f_y}{f_x} \cdot \frac{a^2}{4 \cdot e^2} \cdot \frac{2-A}{C} \right) + \frac{a}{2} \cdot \frac{F_n}{4 \cdot f_y} \cdot \frac{B+2}{C} \quad y_{c2} = 0,005 \text{ m}$$

The angles between the axles and the curve radius will be:

$$\alpha_1 = -\alpha_2 = -\frac{1}{C} \cdot \frac{a}{2 \cdot R} \cdot \frac{180}{\pi} \quad \alpha_1 = -\alpha_2 = -0,008 \text{ grad}$$

The pseudocreep coefficients will have the values:

$$\nu_{1x} = \frac{\gamma}{r} \cdot \left(1 + \frac{f_y}{f_x} \cdot \frac{a^2}{4 \cdot e^2} \cdot \frac{A}{C} \right) \cdot y_{c0} - \frac{e}{R} \quad \nu_{1x} = 0,0008$$

$$v_{ly} = \frac{1}{C} \cdot \frac{a}{2 \cdot R}$$

$$v_{ly} = 0,0001$$

The pseudocreep force should fulfill the relation:

$$T = \frac{f \cdot a}{2 \cdot R \cdot C} \cdot \sqrt{1 + \frac{a^2}{4 \cdot e^2} \cdot A^2} \leq \mu \cdot Q \quad T = 3925 \text{ N} \leq 3985 \text{ N}$$

6. Conclusions

The axles with wear wheel profiles have the advantage of determining a radial position of the leading axle by means of the creep forces involved in the process. The angle α suffers a diminishing transformation, becoming smaller, while the longitudinal elasticity of the guiding axle system increases. Thus, the almost radial position of the axle causes the wear of the rolling surface of the wheels and the rails to reduce its dimension.

It was observed that the increase of the longitudinal elasticity, even useful in curve for allowing the leading axle to come closer to the radial position, it may be quite damaging for the high speed running, having a diminishing effect on the critical speed of the hunting movement. Therefore was studied, in order to determinate the limit values of the longitudinal elasticity.

Moreover, the vehicle may run with small creep forces in curves with considerably smaller radiuses in compared to the rigid guiding based axles.

The conical rolling which diminishes the wear by decreasing the longitudinal creep forces stands for as an example of particular running.

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